



Toppers Village

Complete: सारी Trigonometry का सार

**सारे Trigonometry
फॉर्मूला + TRICKS**

	Numbers
Concepts	6
Formulas	55
Results	67
Graphs	6
TOTAL	134

Watch Complete Free Video on YouTube:

<https://youtu.be/RyXszVxl9ps>



Toppers Village

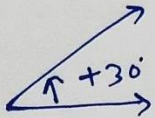
Complete Trigonometry PDF

- सब कुछ
- 6 major concepts (including Angles, minimum, maximum..)
 - 55 formulae
 - 67 याद रखने वाले Results
 - 6 Graphs (with tricks)
-
- 134 Tricks
-

Toppers Village

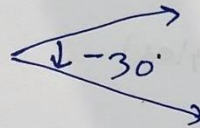
Positive Angles

Anticlockwise



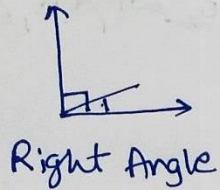
Negative Angles

Clockwise



Degree System (DMS)

Degree minute second.

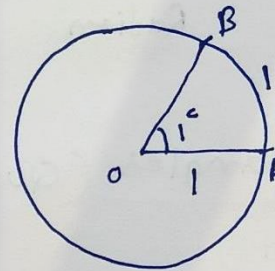


$$1 \text{ Right Angle} = 90 \text{ parts} = 90^\circ$$

$$1^\circ = 60 \text{ parts} = 60' = 60 \text{ minutes}$$

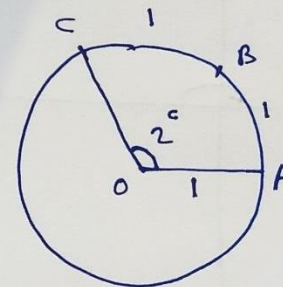
$$1' = 60 \text{ parts} = 60'' = 60 \text{ seconds.}$$

Radian System

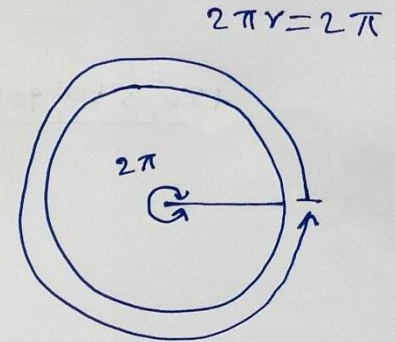


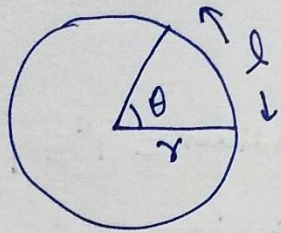
Unit Circle
($r = 1$ unit)

1 radian = $1^c = 1$
= in a unit circle,
angle subtended
by unit arc
at the centre
of circle



$$\# \frac{D}{180} = \frac{R}{\pi} \quad \#$$

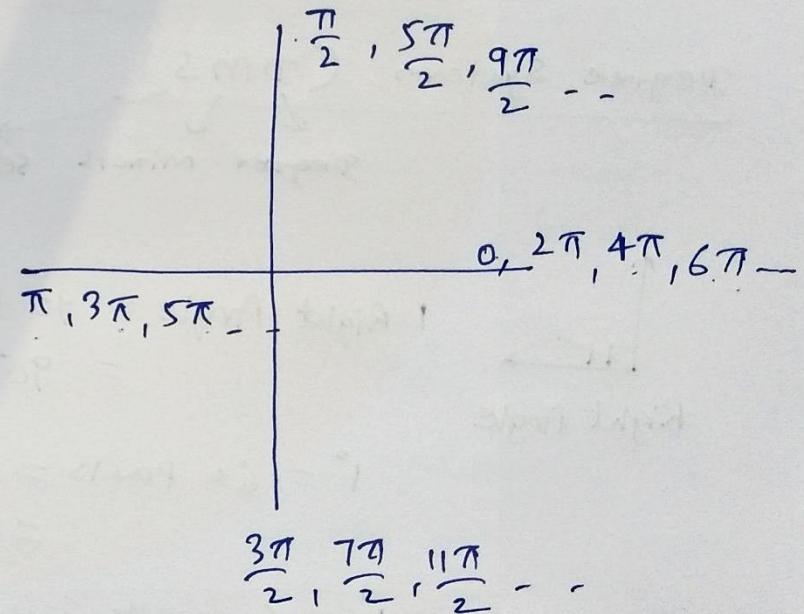
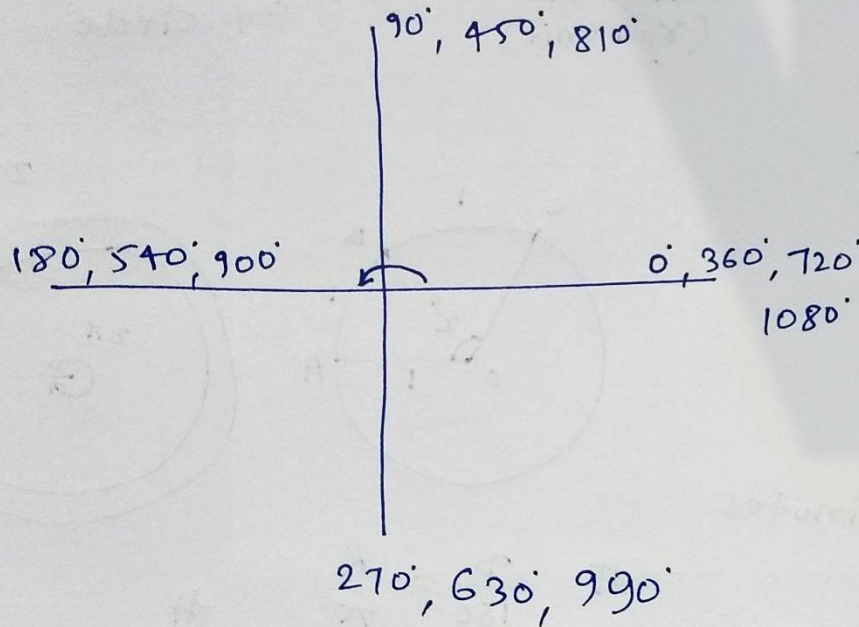


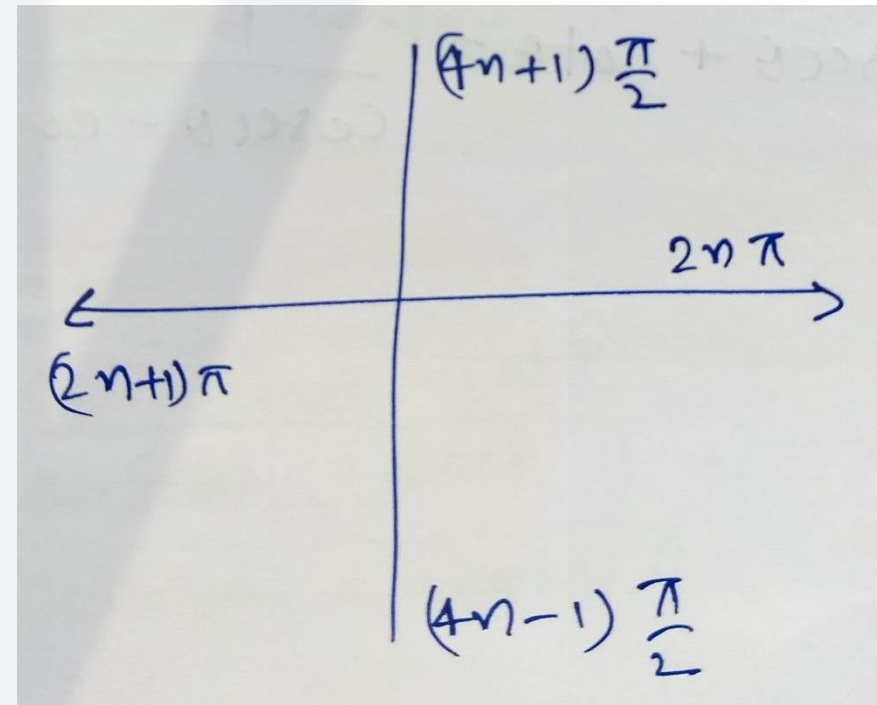
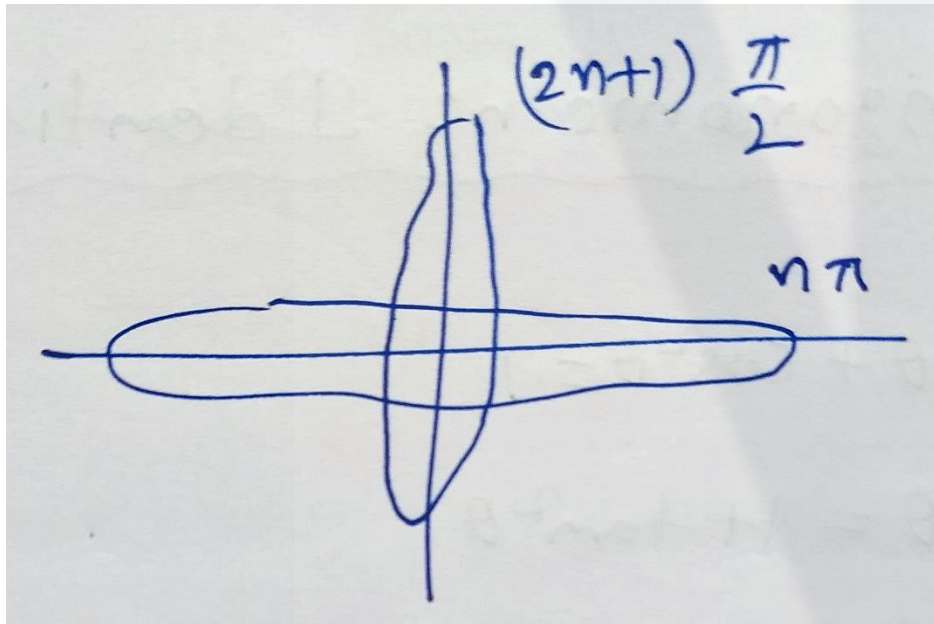


$$l = r\theta$$

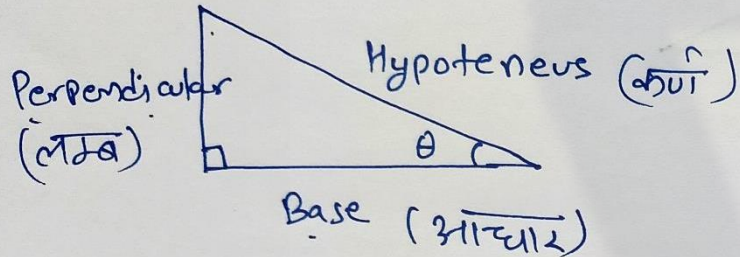
Radian

Pattern of all angles (Quadrantal Angles)



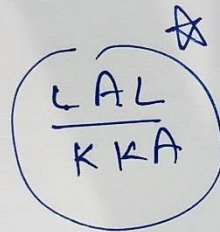


Trigonometry



#

$\sin \theta$	$\cos \theta$	$\tan \theta$
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$
$\text{Cosec } \theta$	$\sec \theta$	$\cot \theta$



#

$$\text{Cosec } \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Trigonometric Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\text{Cosec}^2 \theta = 1 + \cot^2 \theta$$

$$\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

$$\text{Cosec } \theta + \cot \theta = \frac{1}{\text{Cosec } \theta - \cot \theta}$$

Quadrant wise Sign

$\sin (+)$ $\operatorname{cosec} (+)$	$\cos (+)$ $\sec (+)$
$\tan (+)$ $\cot (+)$	

All $(+)$ ve $\Rightarrow$ $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

$\sin(-\theta) = -\sin\theta$

$\boxed{\cos(-\theta) = \cos\theta}$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\boxed{\sec(-\theta) = \sec\theta}$

$\operatorname{cosec}(\theta) = -\operatorname{cosec}\theta$

Angle $\rightarrow x+y, x-y$

$$\begin{aligned}\sin(x+y) &= \sin x \cos y + \sin y \cos x \\ \sin(x-y) &= \sin x \cos y - \sin y \cos x \\ \cos(x+y) &= \cos x \cos y - \sin y \sin x \\ \cos(x-y) &= \cos x \cos y + \sin x \sin y\end{aligned}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

$$\begin{aligned}\tan(x+y+z) &= \frac{\tan x + \tan y + \tan z}{1 - (\tan x \cdot \tan y + \tan y \cdot \tan z + \tan z \cdot \tan x)}\end{aligned}$$



Very important Results.

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec} \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) = \sec \theta$$

$$\sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta$$

$$\cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\cos(\pi - \theta) = -\cos \theta$$

$$\tan(\pi - \theta) = -\tan \theta$$

$$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec} \theta$$

$$\sec(\pi - \theta) = -\sec \theta$$

$$\cot(\pi - \theta) = -\cot \theta$$

Very important Results.

$$\sin(\pi + \theta) = -\sin \theta$$

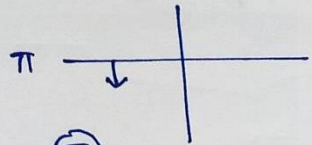
$$\cos(\pi + \theta) = -\cos \theta$$

$$\tan(\pi + \theta) = +\tan \theta$$

$$\operatorname{cosec}(\pi + \theta) = -\operatorname{cosec} \theta$$

$$\sec(\pi + \theta) = -\sec \theta$$

$$\cot(\pi + \theta) = +\cot \theta$$



$$\sin\left(\frac{3\pi}{2} - \theta\right) = ?$$

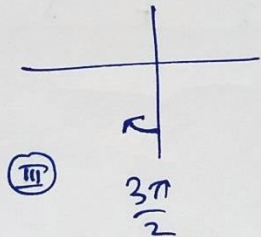
$$\cos\left(\frac{3\pi}{2} - \theta\right) = ?$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = ?$$

$$\operatorname{cosec}\left(\frac{3\pi}{2} - \theta\right) = ?$$

$$\sec\left(\frac{3\pi}{2} - \theta\right) = ?$$

$$\cot\left(\frac{3\pi}{2} - \theta\right) = ?$$



$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos \theta$$

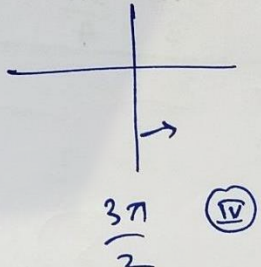
$$\cos\left(\frac{3\pi}{2} + \theta\right) = +\sin \theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot \theta$$

$$\operatorname{cosec}\left(\frac{3\pi}{2} + \theta\right) = -\sec \theta$$

$$\sec\left(\frac{3\pi}{2} + \theta\right) = +\operatorname{cosec} \theta$$

$$\cot\left(\frac{3\pi}{2} + \theta\right) = -\tan \theta$$



$$\sin(2\pi - \theta) = -\sin \theta$$

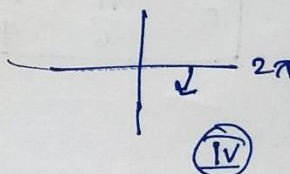
$$\cos(2\pi - \theta) = +\cos \theta$$

$$\tan(2\pi - \theta) = -\tan \theta$$

$$\operatorname{cosec}(2\pi - \theta) = -\operatorname{cosec} \theta$$

$$\sec(2\pi - \theta) = +\sec \theta$$

$$\cot(2\pi - \theta) = -\cot \theta$$



Trigonometric Table.

	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$+\frac{1}{\sqrt{3}}$	+1	$+\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-2	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$	$+\sqrt{3}$	+1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	∞
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
		I				II				III				IV			
		(All)				(S)				(T)				(C)			

$$\# \sin n\pi = 0$$

$$\cos (2n+1) \frac{\pi}{2} = 0$$

$$\tan n\pi = 0$$

$$\cot (2n+1) \frac{\pi}{2} = 0$$

$$\# \begin{cases} \sin(\underline{2n\pi} + u) = \sin u \\ \cos(2n\pi + u) = \cos u \\ \tan(2n\pi + u) = \tan u \\ \tan(n\pi + u) = \tan u \end{cases} \quad n \in \mathbb{I}$$

#

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$$

$$\sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

$$\sin 54^\circ = \frac{\sqrt{5} + 1}{4}$$

$$\sin 72^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4} = \cos 18^\circ$$

$$\begin{aligned}\# \quad \sin x + \sin y &= 2 \sin \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right) \\ \sin x - \sin y &= 2 \sin \left(\frac{x-y}{2} \right) \cdot \cos \left(\frac{x+y}{2} \right) \\ \cos x + \cos y &= 2 \cos \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right) \\ \star \quad \cos x - \cos y &= -2 \sin \left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right)\end{aligned}$$

$$\begin{aligned}\# \quad 2 \sin x \cdot \cos y &= \sin(x+y) + \sin(x-y) \\ 2 \cos x \cdot \sin y &= \sin(y+x) + \sin(y-x) \\ 2 \cos x \cdot \cos y &= \cos(x+y) + \cos(x-y) \\ \star \quad 2 \sin x \cdot \sin y &= \cos(x-y) - \cos(x+y)\end{aligned}$$



Multiple Angles.

$$\begin{cases} \sin 2x = 2 \sin x \cdot \cos x \\ \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \cos 2x = \cos^2 x - \sin^2 x \\ \cos 2x = 2 \cos^2 x - 1 \\ \cos 2x = 1 - 2 \sin^2 x \\ \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \end{cases}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \quad (31-43)$$

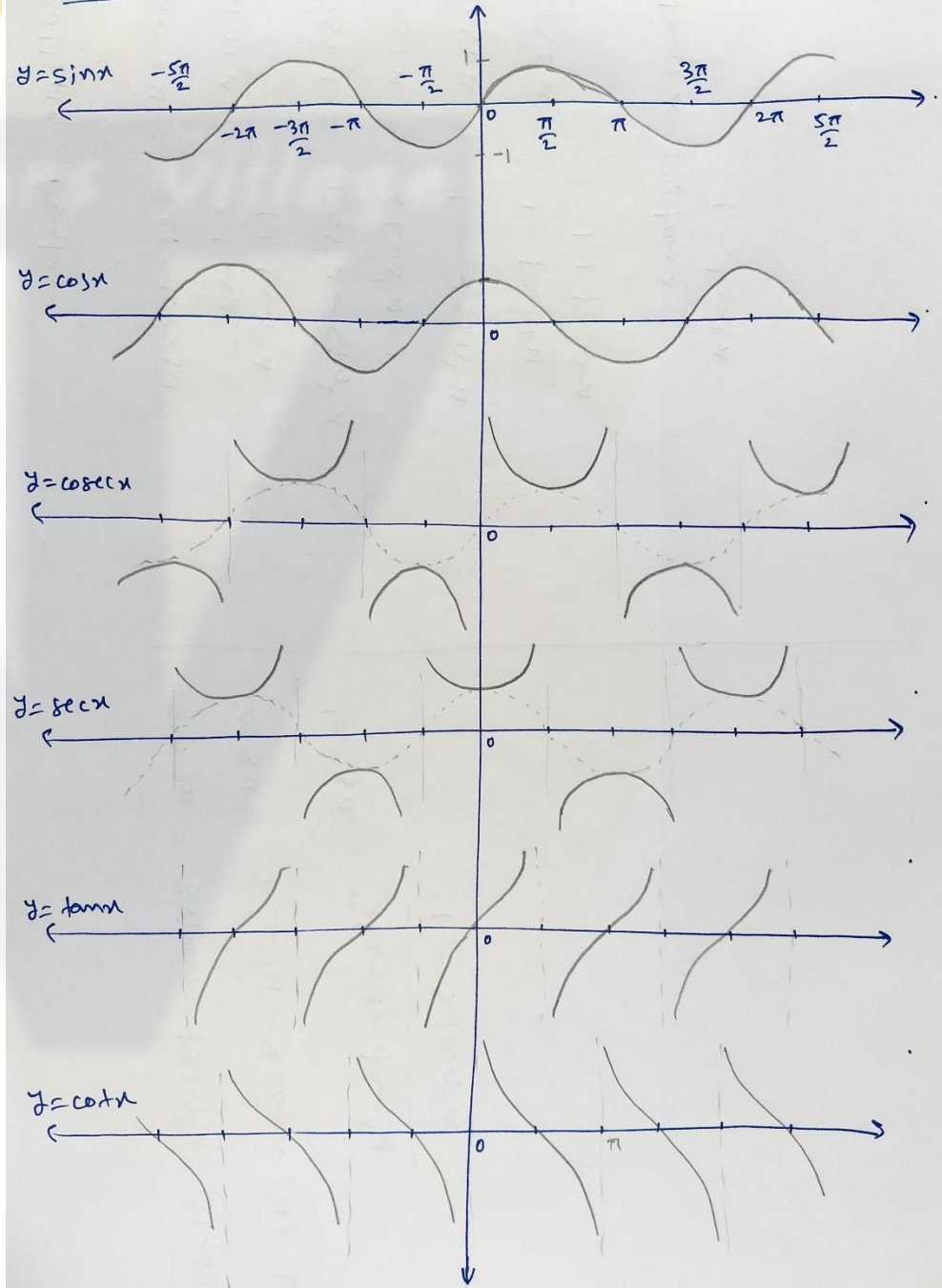
$$\cos 3x = 4 \cos^3 x - 3 \cos x \quad (43-31)$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$



Graphs

Graphs



Minimum & maximum values.

$$-1 \leq \sin u \leq 1$$

$$-1 \leq \cos u \leq 1$$

$$-\sqrt{a^2+b^2} \leq a \cos u + b \sin u \leq \sqrt{a^2+b^2}$$

$$\min. (a \sin^2 u + b \sec^2 u) = 2\sqrt{ab}$$

$$\min. (a \cos^2 u + b \sec^2 u) = \cancel{2\sqrt{2a}} \quad 2\sqrt{ab}$$

$$\min. (a \tan^2 u + b \cot^2 u) = 2\sqrt{ab}$$

Trigonometric Equation.

For example

$$(2 \sin x - 1 = 0)$$

Principal solution $[0, 2\pi)$

General Solution.

$x \rightarrow \text{unknown}$, $\theta = \text{known}$

$$\sin x = \sin \theta \longrightarrow x = n\pi + (-1)^n \cdot \theta \quad \underline{n \in \mathbb{Z}}$$

$$\cos x = \cos \theta \longrightarrow x = 2n\pi \pm \theta$$

$$\tan x = \tan \theta \longrightarrow x = n\pi + \theta$$

$$\left. \begin{array}{l} \sin^2 x = \sin^2 \theta \\ \cos^2 x = \cos^2 \theta \\ \tan^2 x = \tan^2 \theta \end{array} \right\} x = n\pi \pm \theta$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>